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Study of Space Charge Effects in the Fermilab Booster*

Introduction

The emittance growth by space charge in the Booster is considered as one of the bottlenecks limiting the luminosity and the total current of the Tevatron. In fact, the previous study has successfully shown several strong evidences for the effect (Ref.1). First, the transverse emittance increases linearly with the beam current. This suggests that the phase space density has the upper limit predicted by the Laslett formula which is commonly used for the estimation of the tune shift by the space charge. Second, the emittance growth occurs right after the injection and no more later. Since the self-electromagnetic force decreases as the beam energy goes up, space charge effects are strongest near the injection energy. Third, the width of the half integer resonance seems to become wider as the beam intensity increases. This might be due to the large tune spread proportional to the beam intensity.

Probably these results are enough to conclude that the source of the emittance growth in the Booster is space charge. In this study, we tried to confirm these results by using the different measurement system under 200 MeV DC field operation. Specifically, we ask

1. How does the transverse emittance grow up as a function of the beam intensity for the coasting beam?

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2. What is the time evolution of the emittance, especially between the injection and 1 msec later?

Furthermore, if the half integer resonance restricts the maximum tune shift, that is, if there is a certain upper limit of the phase space density, this density limit should be lower when the bare tune is down to the half integer resonance. With this in mind, we ask

3. What is the dependence of the emittance growth on the bare tune?

All the results below were taken without RF since we want to eliminate as many unknown factors as possible, for example the effect of the longitudinal motion and more specifically the bunching factor, and observe space charge effects in the simplest environment. Although it is difficult to make high peak intensity with the coasting beam, the advantage of being able to ignore these factors associated with the complicated RF capture process is considerable.

Experimental Setup

<u>Vertical profile measurement</u>

The measurement of vertical profiles was done by using a single wire and a kicker magnet. The detailed layout of their locations in the Booster is shown in Fig.1 and the lattice function in Fig.2 (Ref.2). The following is one measurement cycle giving a profile. First, the single wire is positioned slightly off the edge of the circulating beam so that the wire does not disturb the beam. At the time we want to measure the profile, say 1 msec after the injection, the kicker magnet is fired. The beam deflected by the kicker hits the wire and the wire produces current. Figure 3 shows the wire signal as a function of time. Because of the finite rising time of the kicker excitation, the beam is moving relative to the wire and the wire signal is changing slowly at the beginning. We interpret that the signal level at the flat top corresponds to the beam intensity at a certain vertical position. Since the beam revolution period is 2.8 μsec, the reading of the signal is actually done within one turn. Then the wire is shifted outward by small amount and we repeat the same thing, firing the kicker and reading the wire signal. This gives us the beam intensity at another position. By repeating this sequence 10 to 20 times, that is, the beam intensity measurement at 10 to 20 different positions in the vertical direction, we can construct the beam profile as shown in Fig.4.

The advantage of the measurement system compared with the flying wire or the profile monitor at the 8 GeV line is that we can measure the beam profile with an excellent time resolution. Since the measurement of the one vertical position takes less than one circulation time, we can in principle set the time of the measurement, for example, at a few turns from the injection. In comparison, the flying wire typically needs about 1.5 msec to fly through the beam. The phenomena which occur faster than this time cannot be detected by the flying wire. For the space charge study, the time resolution is highly desirable. Needless to say, the profile measurement at the 8 GeV line gives us only the overall beam profile in one whole Booster cycle. The disadvantage is that it takes about 30 minutes to construct one profile since the single wire must be positioned at the different place manually. Moreover, within the time period of measuring the whole profile, which is usually 30 minutes or so, the beam shape and size must remain the same. This is sometimes hard to achieve, especially when we choose the tune at critical values, for example, near a resonance. Of course this drawback can be removed if a multiwire system is available.

Definition of the emittance

From the profile data, the vertical emittance can be defined if we know the beta function at the wire position in the following way,

$$\varepsilon = w^2/\beta,$$

where ε is the unnormalized emittance, w is the beam width, and β is the beta function at the wire position. Note that this emittance is not an area of anything (Ref.3). The next question is how to define the width from the profile data. We will define several widths, more strictly half width, of the profile to analyze the data.

Gaussian sigma width

By minimizing the chi-square of the Gaussian fitted curve on the profile data, sigma and its error can be derived. We call this Gaussian sigma width. The curve in

Fig.4 shows the Gaussian fitted on the raw data histogram. We use the program library PAW on the Fermilab VAX cluster to do the chi-square fitting.

38%, 68%, and 95% width

First of all, the mean position and the second moment of the profile are calculated. Based on them, we smoothed out the profile data using the Hermite polynomials with the weighting function, which is Gaussian in this case, taking first 10 terms. Then, the interval from the mean, right-hand side and left-hand side separately, is determined such that the area enclosed by the interval contains 38%, 68%, or 95% of the total area. We call this interval 38%, 68%, or 95% width. Figure 5 shows the profile fitted by Hermite polynomials with the boundary of these widths.

The idea behind these definition is that we would like to see the emittance growth of the core and the tail of the beam by 38% and 95% widths, respectively, and something similar to RMS value by 68% and the Gaussian sigma widths.

<u>Beam intensity measurement</u>

There is always a beam loss immediately after the injection to a few msec so that the number of injection turns, which is the coarse knob to change the intensity in the control room, is not a good measure of the beam intensity. To measure the beam intensity associated with the emittance at certain time, we read the CHG0 plot as shown in Fig.6. The beam intensity drops down sharply when the kicker is fired. We take the intensity level right before the kicker excitation as the beam intensity.

<u>Tune measurement</u>

The horizontal and vertical tunes are measured by Schottky device. The beam is kicked in the transverse direction and provides the oscillation signal of the center of gravity. The signal is processed by digitizing signal analyzer (DSA602). Figure 7 shows the example of the Schottky device signal for the coasting beam with six turn injection. The sharp peaks at the frequency of 361 kHz and its harmonics show the revolution frequency. The frequency interval between one of the sidebands and the revolution frequency should give the fractional part of the tune,

$$n \pm \nu = \Delta f / f_{rev},$$

where n is the nearest integer of the tune, ν is the tune, Δf is the distance between the sideband and the revolution frequency, and f_{rev} is the revolution frequency. To determine the frequency shift of the sidebands accurately, we measure six sidebands associated with three harmonics of the revolution frequency and take the average of them. The ambiguity of the sign is removed by changing the quadrupole strength slightly and checking frequency shift of the sidebands. The observed tune points with several quadrupole strengths are listed in Table.1 and shown in Fig.8.

Table 1: Quadrupole current (Amp.) and the tune

QL	QS	ν_x	ν_y
0.00	0.00	6.704	6.784
0.14	-0.10	6.748	6.822
-0.14	0.10	6.666	6.743
-0.20	0.00	6.728	6.683
-0.56	0.40	6.545	6.621
-0.50	0.00	6.769	6.591
-0.60	0.00	6.782	6.554

From these samples, the relationship between the quadrupole strength and the tune is found by the least square method:

$$QL = 0.6782(\nu_x\text{-}6.70) + 2.7652(\nu_y\text{-}6.78),$$

$$QS = -1.8764(\nu_x\text{-}6.70) - 0.6170(\nu_y\text{-}6.78).$$

Momentum spread measurement

The momentum spread of the incoming beam is measured by observing the debunching time. The vertical kicker is fired and nearly half of the coasting beam is lost. The time structure of the beam in the ring is measured by the resistive wall current monitor and digitizing signal analyzer (DSA602). As time goes on, the portion of the ring occupied by the beam becomes larger. The relationship between the debunching time and the momentum spread is

$$dp/p=(dt/t)/\eta,$$

where η is the slip factor $1/\gamma^2-1/\gamma_t^2$, which is 0.642 for the Booster at 200 MeV. By this method, the momentum spread has been found to be about 0.3%.

Linac output emittance

Figure 9 shows the vertical and horizontal emittance of the linac. The vertical unnormalized 95% emittance is 9.7 π.mm.mrad.

Results

Shape of the profile

Let us take a look at the raw date of the profile monitor first. Figure 10 shows the result of the vertical profile as a function of the total beam intensity when the tune is $(\nu_x,\nu_y)=(6.8,6.7)$. The data is taken 1 msec after the injection. The fitting curve is made by Hermite polynomials up to 10 terms, and it is used to define the 38%, 68%, and 95% widths as described before. Remember that all the results below are taken for the coasting beam.

When the beam intensity is fairly low, the profile looks Gaussian plus some triangular shape at the tail. This looks similar to the profile observed at the downstream of the debuncher in the 200 MeV line (Ref.4). It is reasonable that the beam profile does not change even after the beam is injected into the Booster when the beam intensity is low enough. As the beam intensity increases, the center part of the profile becomes thick and more like parabolic. We can see that the shape is almost unchanged above the intensity $1.79*10^{12}$ and only the area increases to accommodate the higher intensity. In general, the shape looks slightly asymmetric although we have no idea about the reason at present.

Figure 11 shows the profile with the same condition as Fig.10 except for the vertical tune $\nu_y=6.6$. At the lowest intensity, the shape looks almost the same as the case with $\nu_y=6.7$ and it is understandable if the shape represents the incoming beam profile from the linac without any distortion. Below the intensity of $1.46*10^{12}$, the center part of the profile becomes thick in the same way as $\nu_y=6.7$. However, at the intensity of $1.84*10^{12}$ and $2.04*10^{12}$, the shape changes. There seems to be a saturation at the center. At higher intensity, the shape goes back to be more like parabolic again.

Beam intensity vs. emittance

To analyze the profile more quantitatively, we look at the emittance by the several widths. In the following, we use 20 m as the vertical beta function to convert the width to the emittance. All the emittance calculated below is unnormalized value.

Figure 12 shows the emittance defined by the Gaussian sigma. The error bars means the chi-square fitting error of the sigma. Below the intensity of $1.8*10^{12}$, there is a relation between the emittance and the intensity similar to the one found in the previous study (Ref.1). Namely, when the intensity is low enough, the emittance is almost constant. As the intensity exceeds a certain threshold value, there is more or less a linear relation between two quantities. If we draw a straight line according to the linear relation, the line would nearly cross the origin. Below $1.8*10^{12}$, it is hard to tell the tune dependence. However, near about $2*10^{12}$, the emittance has a sharp peak at ν_y=6.6 and goes down again at $2.3*10^{12}$. In fact, this corresponds to the saturated profile we looked at in the previous section. We might be able to attribute the peak to some sort of resonance. For ν_y=6.7, the linear relation continues up to $2.3*10^{12}$. Above this intensity, the emittance is constant, probably because of the aperture limit.

The same sort of picture appears if we define the emittance by the 68% width as shown in Fig.13. The peak of the emittance near $2*10^{12}$ exists and there is no tune dependence except for the peak. However, the flat portion in the low intensity side almost disappears. Moreover, although there is the linear relation below $1.8*10^{12}$, an offset of the emittance can be seen clearly if we extend the straight line to the left.

The emittance defined by the 38% width looks somewhat different. In Fig.14, the tune dependence emerges. The peak of the emittance is still there and it is more exaggerated. The emittance offset, the intersection of the straight line by the linear relation and the y-axis, is finite.

The plot comes out quite differently if we take 95% width. In Fig.15, we can see an almost constant emittance below the intensity of $1.5*10^{12}$ and another constant value above that. The peak of the emittance is reduced. The tune dependence becomes clearer but it is not significant.

Time evolution of the emittance

Figure 16 shows the profile at different time from the injection. The intensity is about $2*10^{12}$ and the tune $(\nu_x,\nu_y)=(6.8,6.7)$. There is little change in the shape during this time period. If we look at the emittance plot by Gaussian sigma, we can see the constancy or a slight decrease of the emittance. This is also true for the emittance defined by 38%, 68%, and 95% widths.

Summary and Discussion

The intensity dependence of the emittance looks similar to the previous study when we define the emittance by Gaussian sigma. Let us suppose the beam distribution is Gaussian even when the intensity is high and the emittance growth comes simply from the phase space density limit. From Fig.12, we approximately get the ratio of the total particle number to the unnormalized RMS emittance,

$$n_t/\varepsilon_{RMS} = 1.1*10^{18}.$$

In terms of the Laslett tune shift,

$$\Delta\nu_y = -\frac{r_p}{4\pi\beta^2\gamma^3}\cdot\frac{n_t}{\varepsilon_{RMS}},$$

the limiting phase space density corresponds to $\Delta\nu_y=-0.23$. It looks reasonable. Considering that the vertical tune is lower than 6.85 which has $\Delta\nu_y=-0.37$ in the previous study, it seems that we reproduced exactly the same results as before.

However, we should notice that we had a big assumption to derive this result. As we have seen before, the shape of the profile itself is a function of the intensity. There is some doubt as to whether we can assume constant distribution to calculate the phase space density. To take into account the change of the distribution, we tried to define three other emittances which physically correspond to the core, RMS, and the tail of the beam. Of course, if the distribution is always the same or can be assumed to be the same, the plot of these emittance looks the same except for the absolute value. This is not the case. In particular, the emittance defined by 95% width looks different from the others. In addition, although there is a linear relation between the emittance and the intensity in 38% and 68% emittance, it is not the constant phase space density line since it does not go through the origin.

We should emphasize here the fundamental difficulty in observing such transient phenomena as space charge effects. The emittance, for example, which we always think as something well defined and easily observed, at least with proper diagnostics, is not as useful a representation of the beam quality as it is supposed to be. The various definitions of the emittance gave us some information, but they are not enough for us to derive a definite conclusion.

The time evolution of the emittance was quite interesting. Although it is impossible to separate the beam loss and the emittance growth, and again a question on the meaning of the emittance might be raised, at least we can say that the final beam distribution is determined at the very early stage, right after the injection.

The tune dependence was not clear. There is a resonance-like structure of the emittance when ν_y=6.6, but the detailed study on this is beyond the scope of this experiment.

Acknowledgments

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3. S.Ohnuma, "The Beam Emittance", EXP-111, November 28, 1983.

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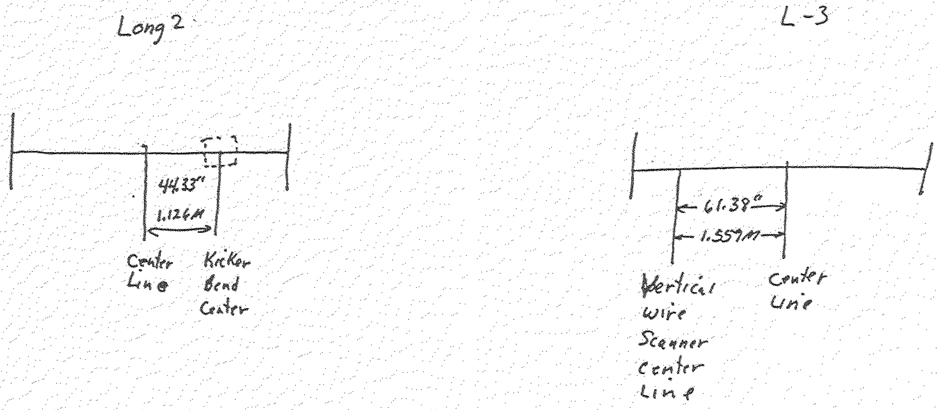


Fig.1: The location of the kicker magnet in the long straight section #2 and the single wire in the long straight section #3.

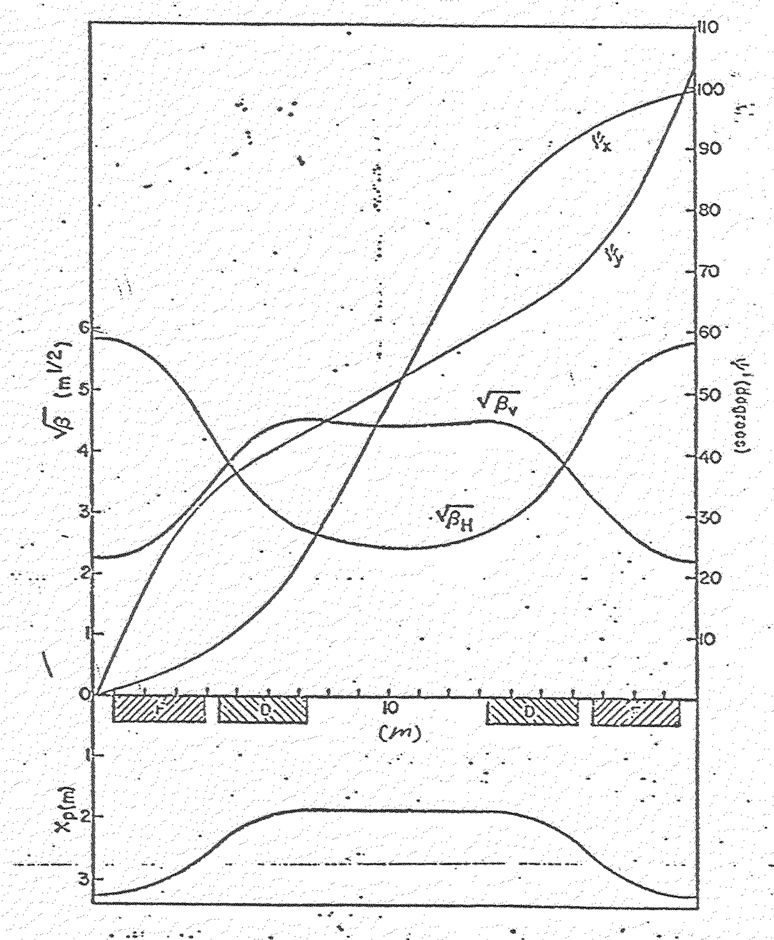


Fig.2: The lattice function of the Booster. This figure is copied from Ref.2.

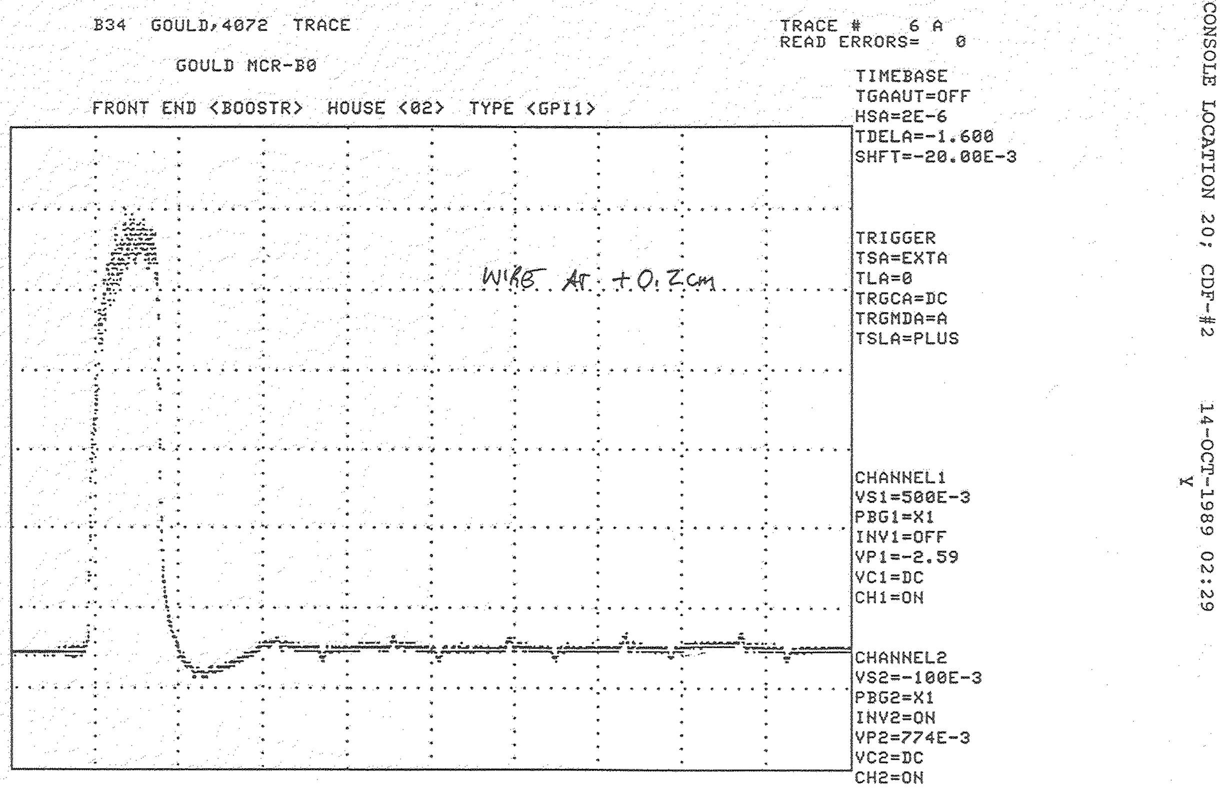


Fig.3: Scope display of the wire signal, 2 μsec/div. in x-axis.

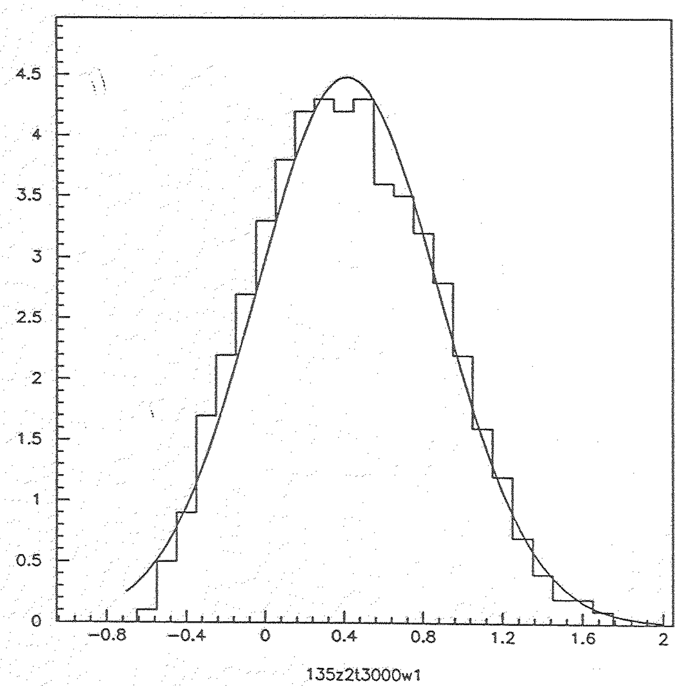


Fig.4: Typical profile measured by the single wire. The x-axis is the wire position and the y-axis is the current induce at the wire by the beam. The curve represents the Gaussian fitted to the raw data.

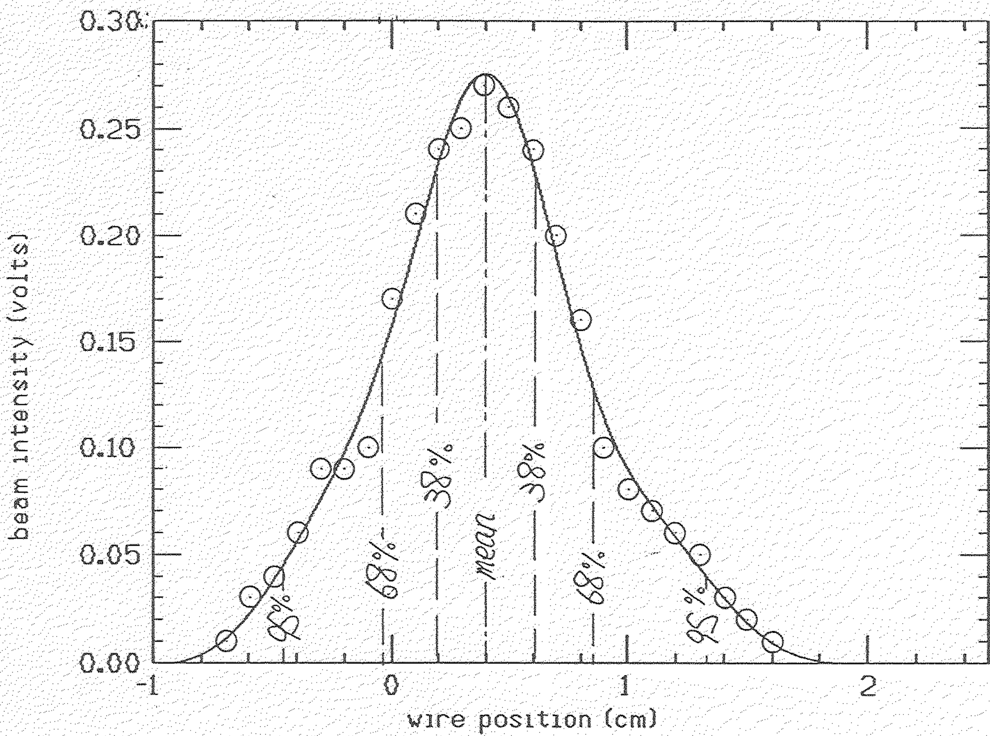


Fig.5: Typical profile and the curve fitted by Hermite polynomials. This also shows the boundary which contains the 38%, 68%, and 95% area.

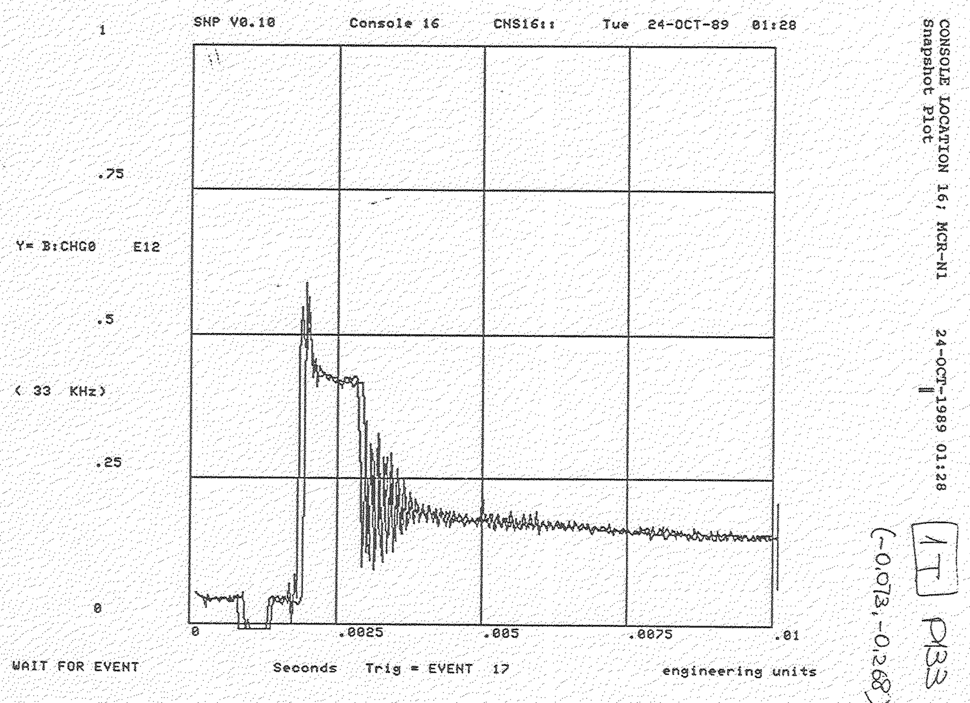


Fig.6: CHG0 plot for the beam of one turn injection. At 1 msec after the injection, the kicker magnet is fired and almost half the beam is lost. We read the intensity right before the kicker excitation.

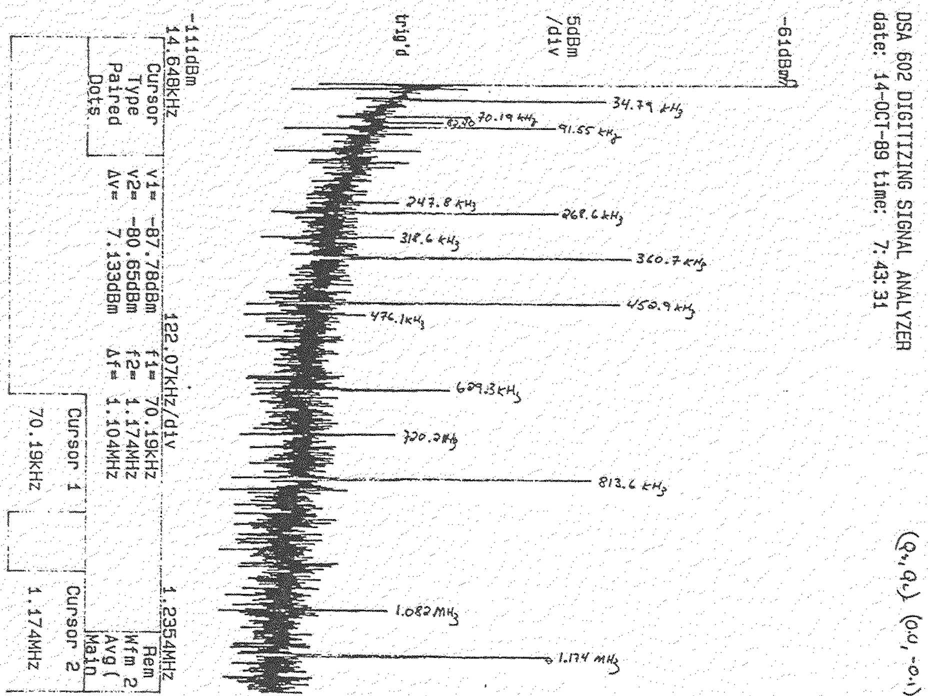


Fig.7: Vertical signal in the six turn coasting beam. The beam revolution frequency is 361 kHz. We can see the sideband of the betatron frequency and some spurious.

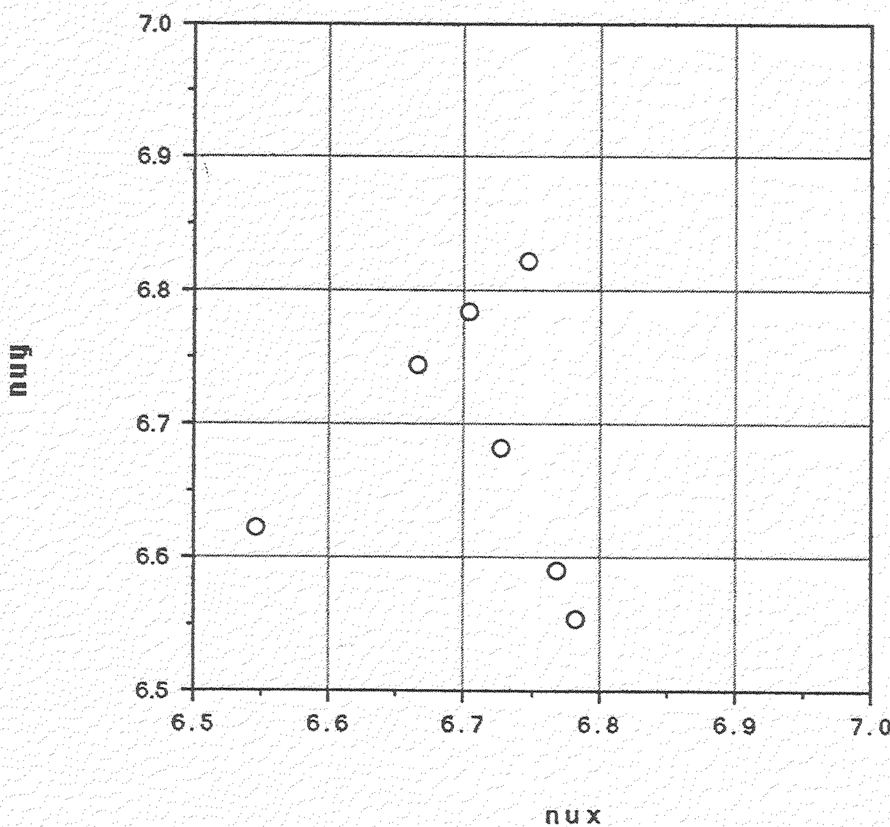
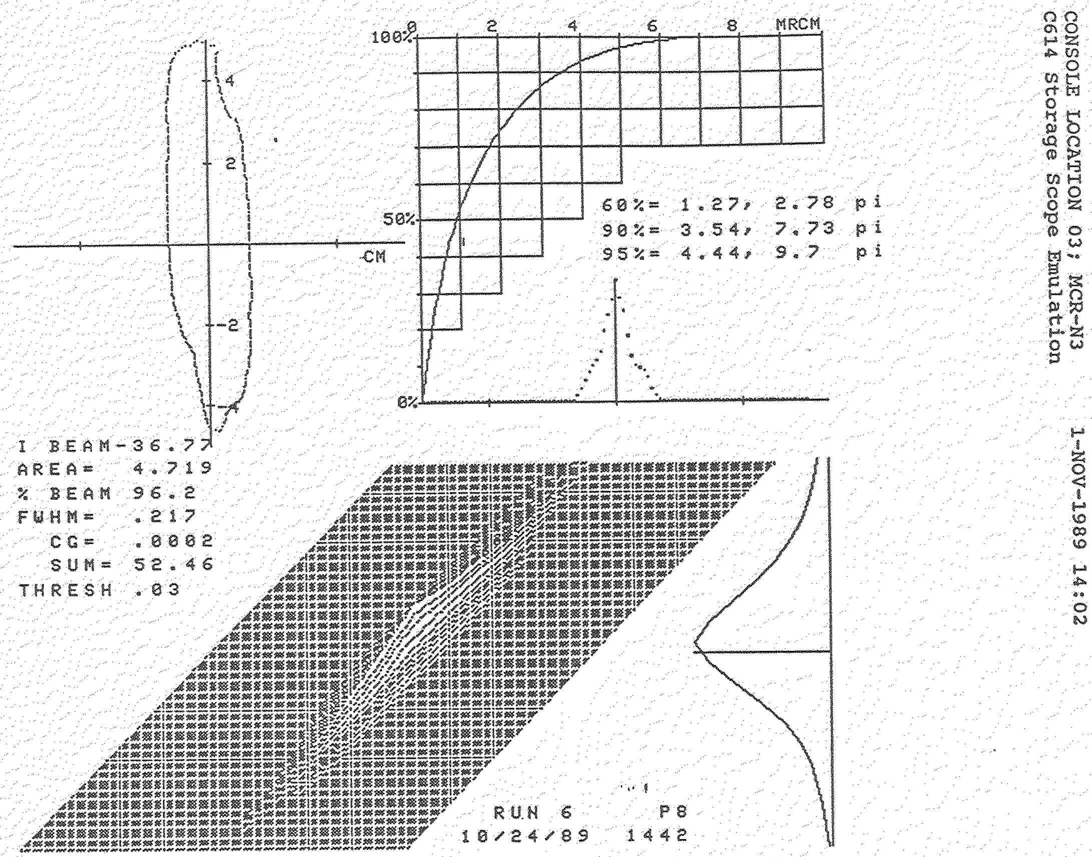


Fig.8: Measured tune of seven set of quadrupole strength. Refer table 1.



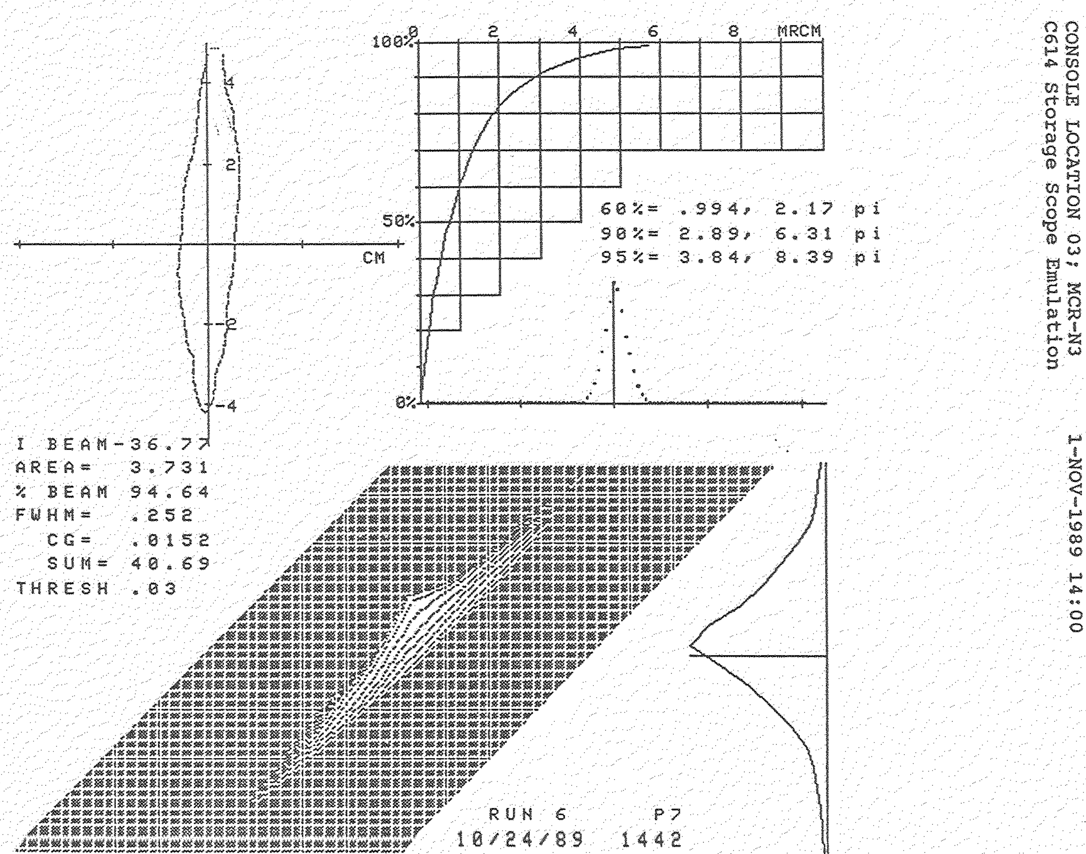


Fig.9: Vertical (above) and Horizontal (below) emittance of the linac.

Unnormalized 95% emittance is 9.7 π mm mrad.

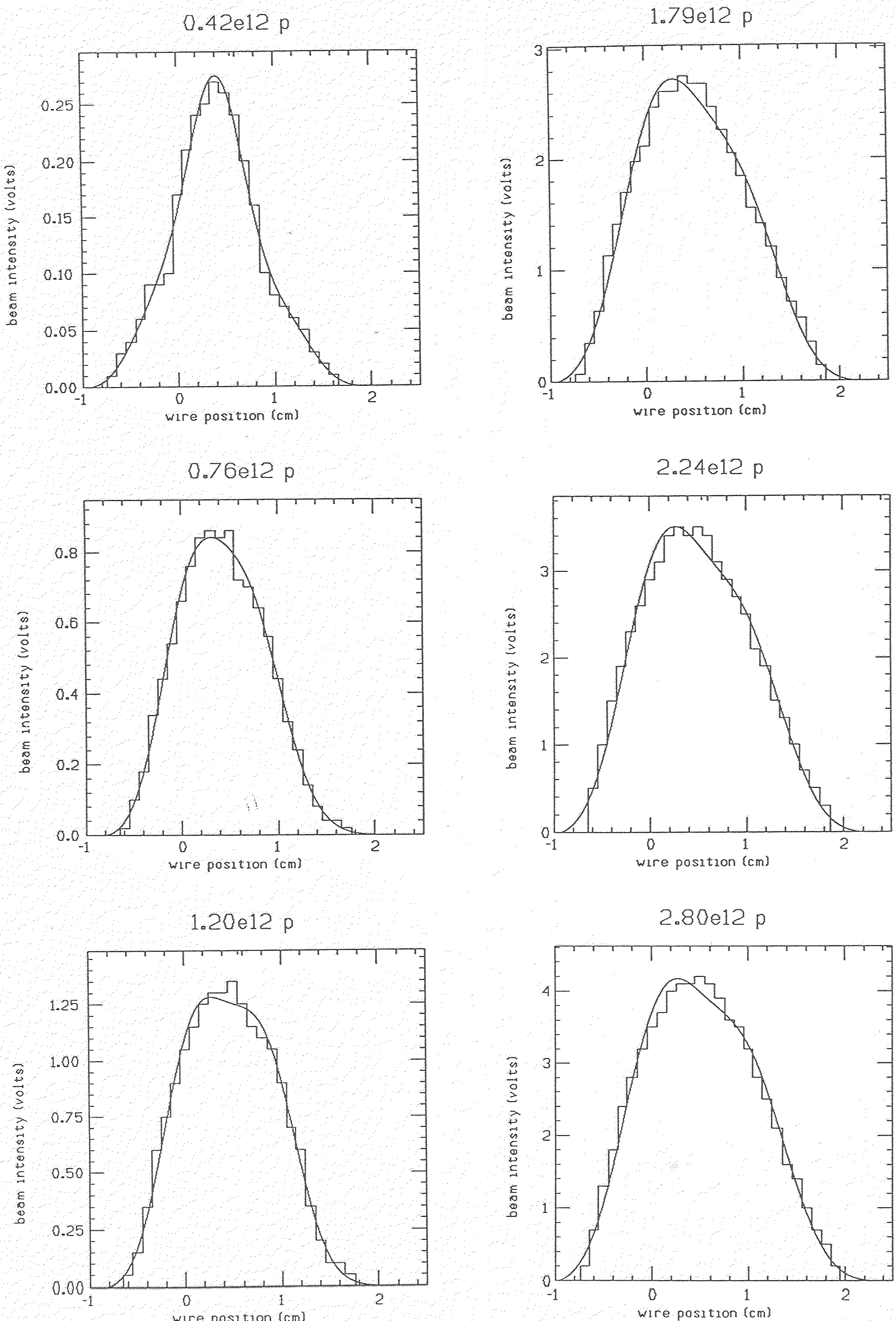


Fig.10: Raw data of the profile and the fitted curve by Hermite polynomials of several intensity 1 msec after injection. The tune is $(\nu_x,\nu_y)=(6.8,6.7)$.

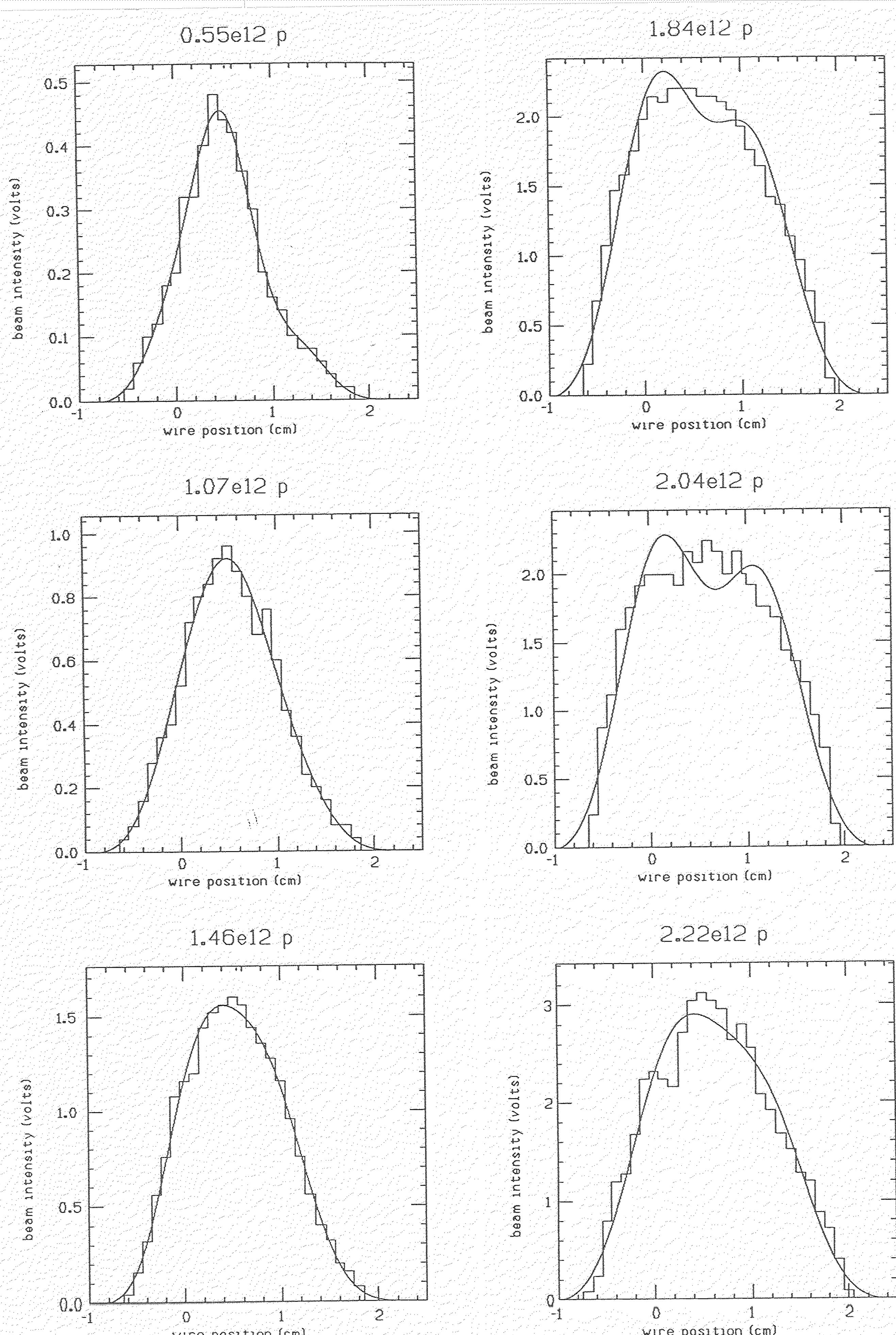


Fig.11: Raw data of the profile and the fitted curve by Hermite polynomials of several intensity 1 msec after injection. The tune is (ν_x,ν_y)=(6.8,6.6).

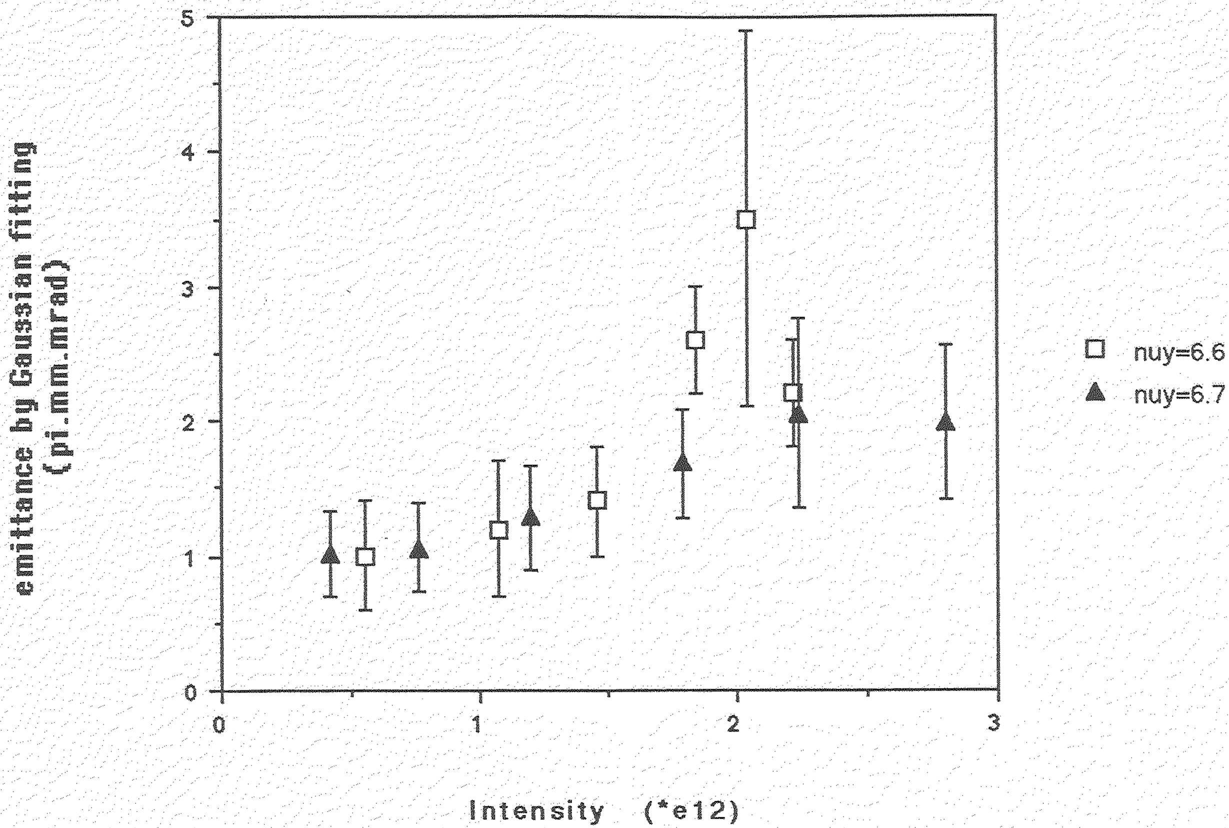


Fig.12: The emittance 1 msec after injection as a function of the beam intensity. The emittance defined by the sigma of the Gaussian fitted by minimizing chi-square.

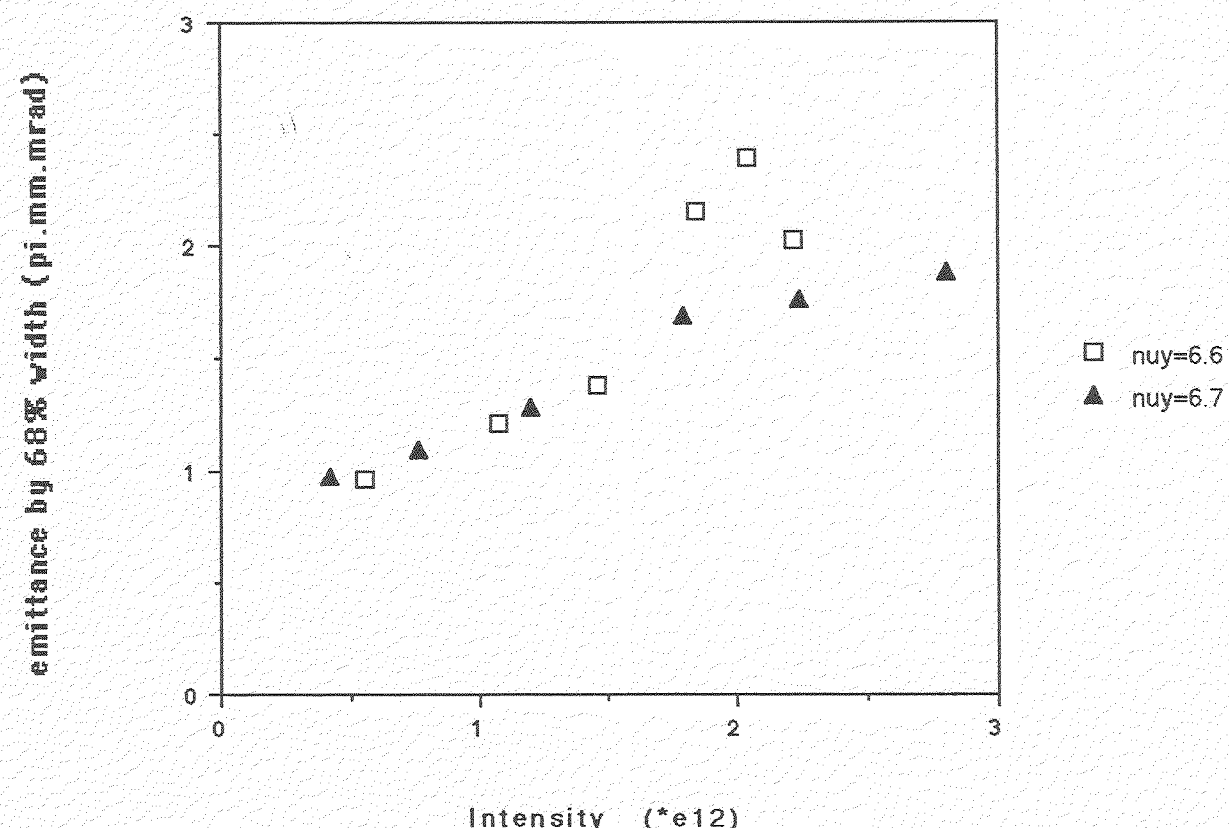


Fig.13: The emittance defined by the 68% area of the Hermite polynomial curve as shown in Fig.5.

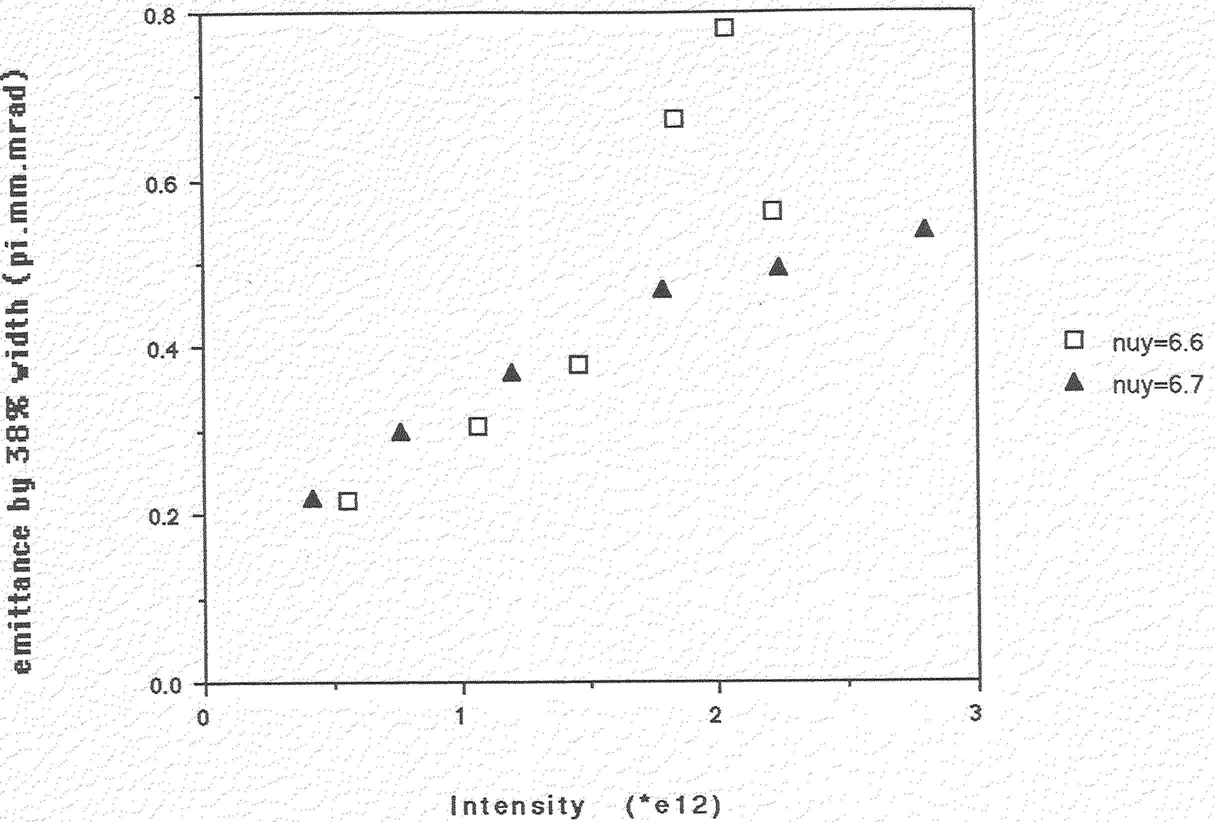


Fig.14: The emittance defined by the 38% area of the Hermite polynomial curve as shown in Fig.5.

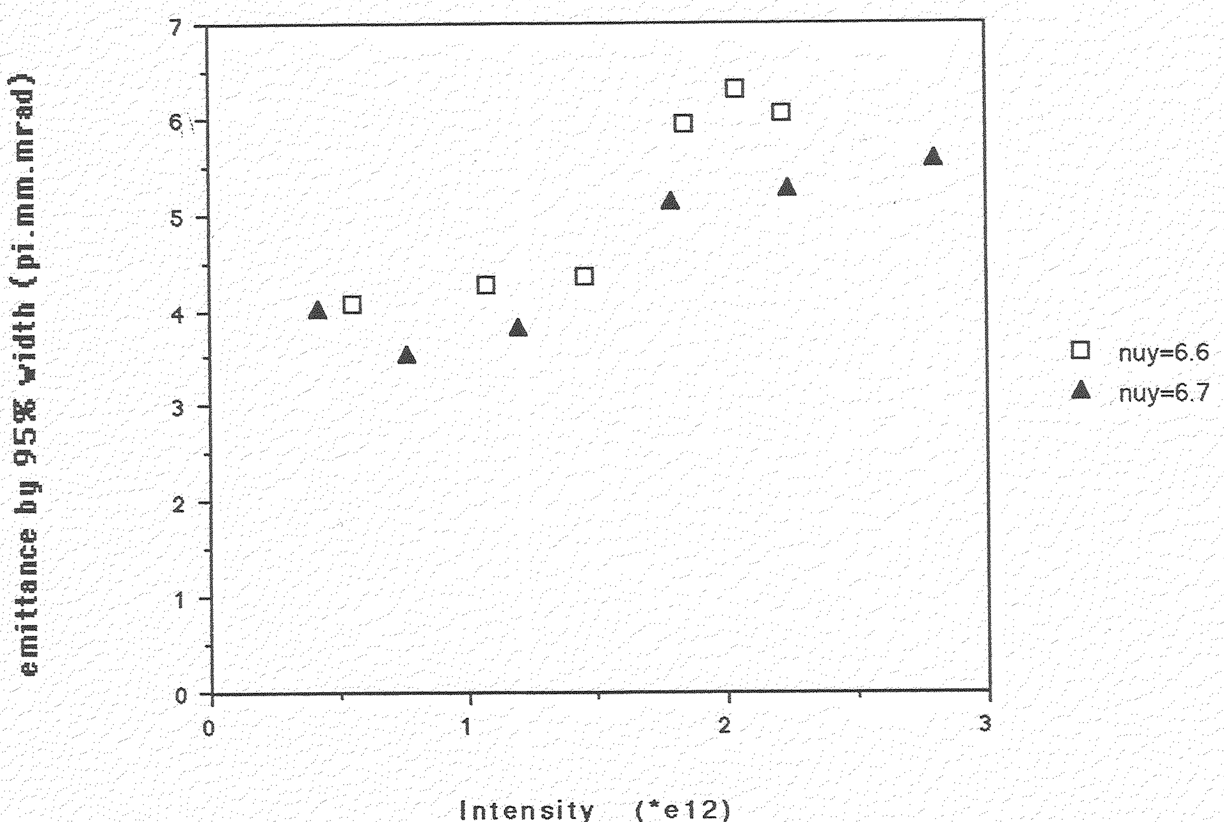


Fig.15: The emittance defined by the 95% area of the Hermite polynomial curve as shown in Fig.5.

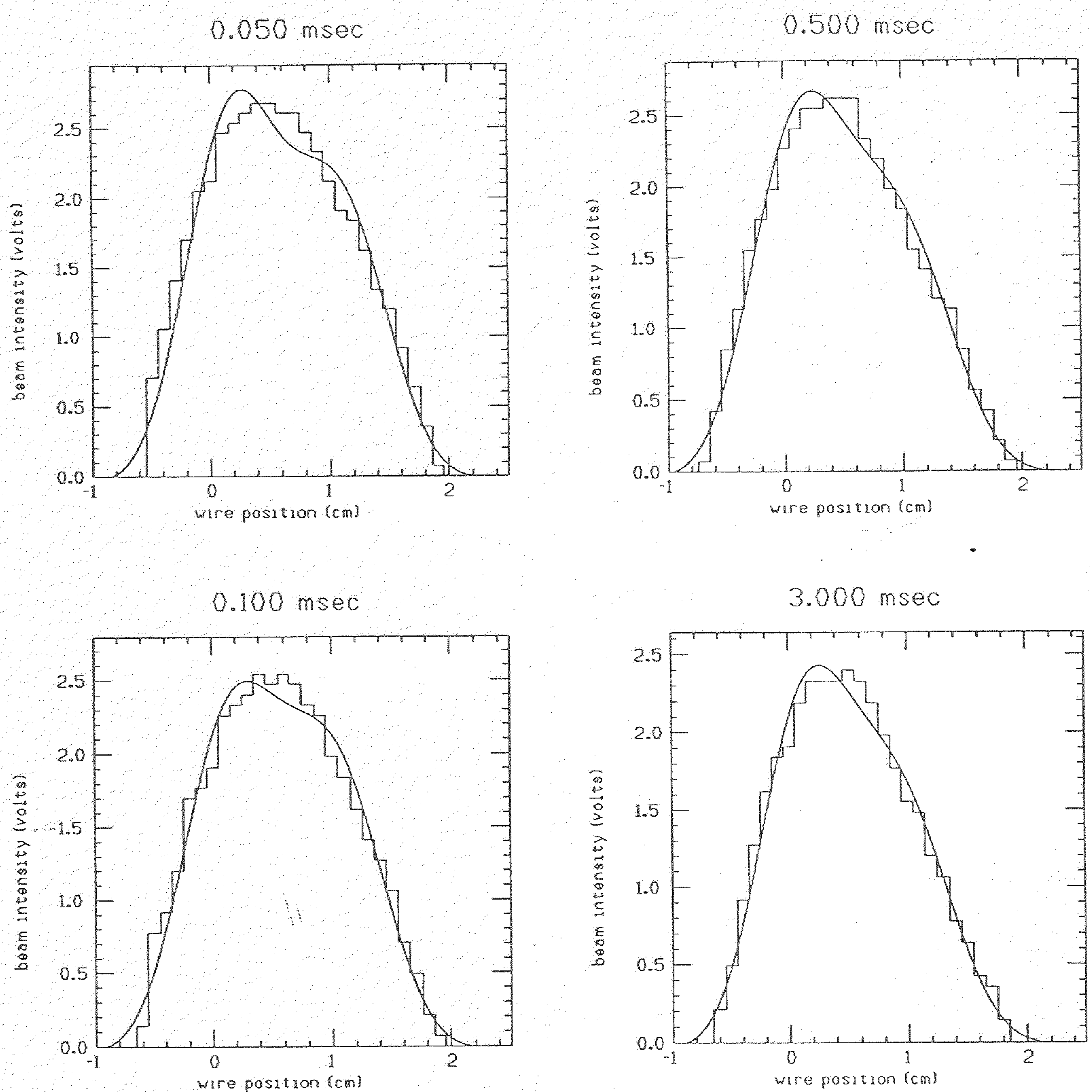


Fig.16: Raw date of the beam profile when the tune is $(\nu_x,\nu_y)=(6.8,6.7)$ and the intensity is about $2*10^{12}$ as a function of time after injection.

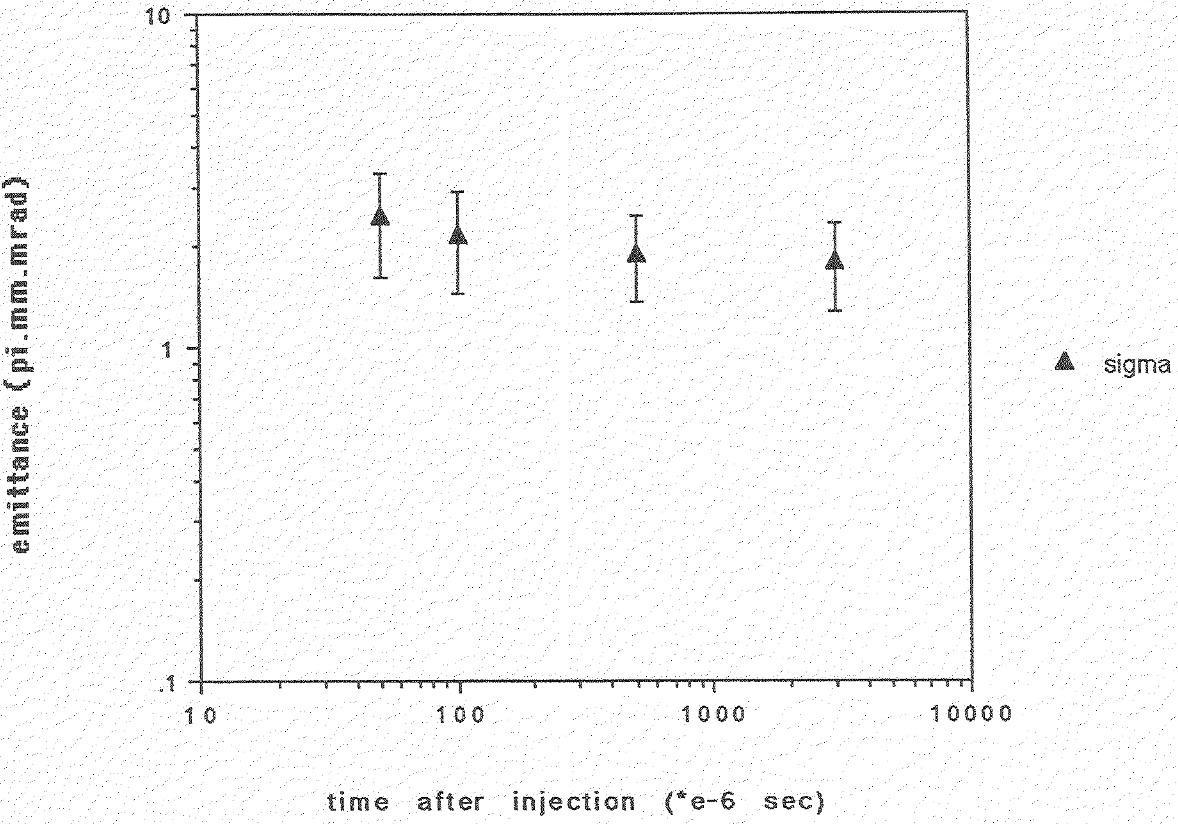


Fig.17: The time evolution of the emittance defined by Gaussian sigma.

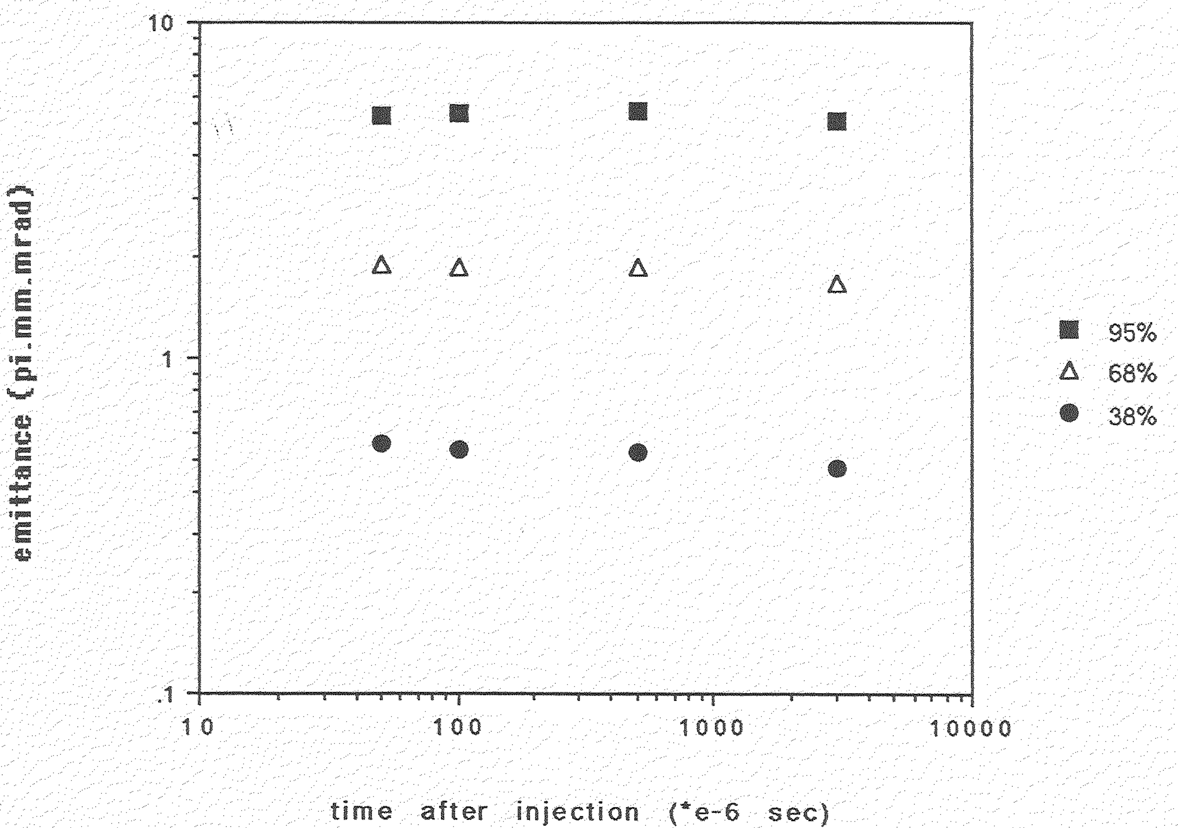


Fig.18: The time evolution of the emittance defined by 38%,68%, and 95% area.